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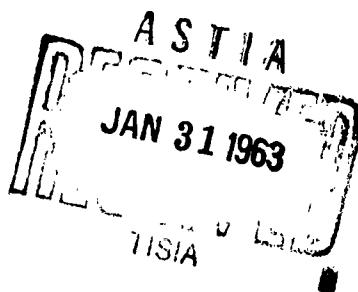
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MACROSCOPIC EQUATIONS GOVERNING THE
INTERACTION OF ELECTROMAGNETIC WAVES
WITH NON-UNIFORM PLASMA FLOWS

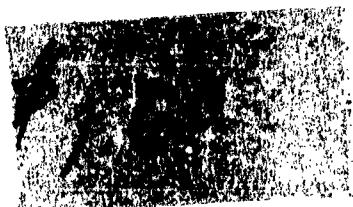


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AEROPHYSICS OPERATION

MACROSCOPIC EQUATIONS GOVERNING THE INTERACTION
OF ELECTROMAGNETIC WAVES WITH NON-UNIFORM
PLASMA FLOWS

By

K. T. Yen

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MISSILE AND SPACE DIVISION

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~~ABSTRACT~~

A method of analysis for the interaction between electromagnetic waves and a non-uniform plasma flow is presented. The electromagnetic waves are taken to be in the microwave range. By using a fully-ionized gas as a model, two coupled sets of macroscopic equations are obtained: the flow equations and the field equations. Special consideration is given to the case where diffusion and charge separation can be disregarded. It is shown that both sets of equations can be reduced to useful forms. Physical significance of these equations and extension of this method to include charge separation and partially ionized plasmas are briefly discussed.

CONTENTS

	PAGE
ABSTRACT	ii
I. INTRODUCTION	1
II. METHOD OF ANALYSIS AND BASIC EQUATIONS - TWO-FLUID MODEL	3
III. REDUCTION OF THE FIELD EQUATIONS	8
IV. FLOW EQUATIONS WITHOUT THE EFFECTS OF DIFFUSION AND CHARGE SEPARATION	10
V. SOME GENERAL CONSIDERATIONS	12
VI. CONCLUDING REMARKS	14
VII. ACKNOWLEDGEMENTS	15
VIII. REFERENCES	16

MACROSCOPIC EQUATIONS GOVERNING THE INTERACTION
OF ELECTROMAGNETIC WAVES WITH NON-
UNIFORM PLASMA FLOWS

I. INTRODUCTION

Propagation of electromagnetic waves in a plasma has been studied by many investigators, but generally the physical properties of the plasma is taken as fixed or even uniform, only slightly modified by the interactions. Thus, the mathematical problem is a linear one. On the other hand, studies by thermo-nuclear physicists of plasma heating deal with quiescent plasmas and attentions are generally directed to the energy absorption and exchange processes.

In this study, a non-uniform plasma is considered and the field strength may be large enough to produce strong interactions which, in turn, control the propagation of the electromagnetic waves and determine the flow field and the state of the plasma gas. The wavelength of the external electromagnetic waves is assumed to be in the microwave range.

This analysis is limited to a study of the quasi-steady case; i. e., it is applicable to the case where the flow field of the plasma is a steady one in the absence of electromagnetic waves and the flow quantities and the physical properties of the plasma reach a quasi-steady state after the electromagnetic field has acted on the plasma for a sufficiently long time. By using the macroscopic equations of motion and the Maxwell equations, a method of analysis is developed in this paper which yields two

sets of differential equations. The first set of equations is time-independent and consists of the continuity, momentum, and energy equations governing the motion of the plasma. The second set of equations governs the wave propagation. These two sets of equations are coupled. In obtaining these equations, however, several approximations are made in order to reduce them to manageable forms.

Application of these equations to several problems are being carried out and will be reported later. It is seen that, in addition to the familiar difficult problem of treating electromagnetic wave propagation in a non-uniform medium, the non-linearity of the flow equations adds to the complexity of the interaction problem.

II. METHOD OF ANALYSIS AND BASIC EQUATIONS - TWO-FLUID MODEL

The method of analysis and the basic equations will be developed based on a two-fluid model, namely, the plasma is fully ionized and consists of protons and electrons. Extension to other cases can be done without any essential difficulty and will be indicated later.

The macroscopic equations governing the motion of the plasma are assumed to have the following form (ref. 1)

$$\frac{\partial n_\alpha}{\partial t} = -\text{div}(n_\alpha \bar{V}_\alpha), \quad (1)$$

$$m_\alpha n_\alpha \frac{D\bar{V}_\alpha}{Dt} = -\nabla p_\alpha + (\text{div} \pi_\alpha) + \frac{e_\alpha}{c} n_\alpha (\bar{E} + \bar{V}_\alpha \times \bar{B}) + \bar{R}_\alpha \quad (2)$$

$$\begin{aligned} \frac{\partial}{\partial t} \left(\frac{1}{2} m_\alpha n_\alpha V_\alpha^2 + \frac{\gamma}{\gamma-1} k n_\alpha T_\alpha \right) &= -\text{div} \left\{ \bar{V}_\alpha \left(\frac{1}{2} m_\alpha n_\alpha V_\alpha^2 + \frac{\gamma}{\gamma-1} k n_\alpha T_\alpha - \pi_\alpha \right) \right. \\ &\quad \left. + \bar{q}_\alpha \right\} + \frac{e_\alpha}{c} n_\alpha \bar{V}_\alpha \cdot \bar{E} - \bar{V}_\alpha \cdot \bar{R}_\alpha + Q_\alpha \end{aligned} \quad (3)$$

$$p = k n_\alpha T_\alpha \quad (4)$$

where $\alpha = e$ or i referring to protons or electrons, respectively, $e_i = e = -e_e$, π_α is the viscous stress tensor, \bar{q}_α is the heat conduction rate per unit volume, \bar{R}_α is the momentum transferred to the α particles by collisions with particles of the other species, Q_i is the heat addition rate per unit volume from the electrons to the protons, Q_e is given by

$$Q_e = \bar{R}_e \cdot (\bar{V}_i - \bar{V}_e) - Q_i \quad (5)$$

and other symbols have their usual meanings. All electromagnetic quantities are in electromagnetic units.

The external electromagnetic field will be assumed to have the form $\exp\{i(\omega t - kx)\}$, and no steady electric or magnetic field is applied. Because of the interactions between the electromagnetic waves and the plasma, dispersion of the waves will occur. It is reasonable to expect that there will be periodic oscillations of the physical and flow properties of the plasma with respect to their time-averaged mean values. A basic assumption will be made that the number density fluctuations are small compared to their respective mean values at every point in the flow field, and may be omitted from the dynamical equations. This is believed to be valid for sufficiently large plasma densities and for values of ω in the microwave range. However, the velocity fluctuations, in particular, those of the electrons, cannot be ignored on the same ground.

The mean value of a physical or flow quantity A is defined by

$$\langle A \rangle = \frac{1}{\tau_0} \int_t^{t+\tau_0} A dt$$

where τ_0 is a properly chosen time interval, e.g., in the present problem, any multiple of the periodicity of A . The requirement that $\langle A \rangle$ be independent of t can be regarded as a formal definition of A being quasi-steady. The fluctuating part of A , denoted by A' , is given by

$$A = \langle A \rangle + A'.$$

Evidently, $\langle A' \rangle = 0$ and $\langle \partial A / \partial t \rangle = \partial \langle A \rangle / \partial t$.

Time averaging of the above macroscopic equations yields the following equations:

$$\operatorname{div}(n_\alpha \bar{V}_\alpha) = 0, \quad (6)$$

$$m_\alpha n_\alpha (\bar{V}_\alpha \cdot \nabla \bar{V}_\alpha + \bar{V}'_\alpha \cdot \nabla \bar{V}'_\alpha) = -\nabla p_\alpha + (\text{div } \pi_\alpha) + \frac{e_\alpha}{c} n_\alpha \langle \bar{V}'_\alpha \times \bar{B} \rangle + \bar{R}_\alpha , \quad (7)$$

$$\text{div} \left\{ \bar{V}_\alpha \left[\frac{1}{2} m_\alpha n_\alpha (V_\alpha^2 + \langle V'_\alpha \rangle^2) + \frac{\gamma}{\gamma-1} k n_\alpha T_\alpha - \pi_\alpha \right] + \bar{q}_\alpha \right\} = \frac{e_\alpha}{c} n_\alpha \langle \bar{V}'_\alpha \cdot \bar{E} \rangle - \bar{V}_\alpha \cdot \bar{R}_\alpha - \langle \bar{V}'_\alpha \cdot \bar{R}'_\alpha \rangle + Q_\alpha , \quad (8)$$

$$p_\alpha = k n_\alpha T_\alpha , \quad (9)$$

where, and also hereafter, the symbols without primes denote the mean values, while the primed symbols denote the fluctuations. In obtaining Eq. (6), the term $\langle n'_\alpha \bar{V}'_\alpha \rangle$ is neglected in view of the assumption that in all dynamic equations the number density fluctuations are disregarded.

By using Eq. (5), the energy equation (8) can be written in the following form:

$$\text{div} \left\{ \bar{V}_i \left[\frac{1}{2} m_i n_i (V_i^2 + \langle V'_i \rangle^2) + \frac{\gamma}{\gamma-1} k n_i T_i - \pi_i \right] + \bar{q}_i \right\} = \frac{e}{c} n_i \langle \bar{V}'_i \cdot \bar{E} \rangle - \bar{V}_i \cdot \bar{R}_e - \langle \bar{V}'_i \cdot \bar{R}'_e \rangle + Q_i , \quad (10)$$

$$\text{div} \left\{ \bar{V}_e \left[\frac{1}{2} m_e n_e (V_e^2 + \langle V'_e \rangle^2) + \frac{\gamma}{\gamma-1} k n_e T_e - \pi_e \right] + \bar{q}_e \right\} = -\frac{e}{c} n_e \langle \bar{V}'_e \cdot \bar{E} \rangle + \bar{V}_i \cdot \bar{R}_e + \langle \bar{V}'_i \cdot \bar{R}'_e \rangle - Q_i , \quad (11)$$

Eqs. (6) to (11) will be called the "flow equations".

By subtracting Eqs. (6) to (9) from Eqs. (1) to (4), respectively, the following system of equations is obtained:

$$\frac{\partial n'_\alpha}{\partial t} + \text{div} (n_\alpha \bar{V}'_\alpha) = 0 , \quad (12)$$

$$\begin{aligned}
& m_\alpha n_\alpha \left(\frac{\partial \bar{V}'_\alpha}{\partial t} + \bar{V}_\alpha \cdot \nabla \bar{V}'_\alpha + \bar{V}'_\alpha \cdot \nabla \bar{V}_\alpha + \bar{V}'_\alpha \cdot \nabla \bar{V}'_\alpha - \langle \bar{V}'_\alpha \cdot \nabla \bar{V}'_\alpha \rangle \right) \\
& = - \nabla p'_\alpha + (\text{div } \pi'_\alpha) + \frac{e_\alpha}{c} n_\alpha \bar{E} + \frac{e_\alpha}{c} n_\alpha (\bar{V}_\alpha \times \bar{B}) \\
& + \frac{e_\alpha}{c} n_\alpha [\bar{V}'_\alpha \times \bar{B} - \langle \bar{V}'_\alpha \times B \rangle] + \bar{R}'_\alpha , \tag{13}
\end{aligned}$$

$$\begin{aligned}
\frac{\partial}{\partial t} (m_\alpha n_\alpha \bar{V}_\alpha \cdot \bar{V}'_\alpha + \frac{\gamma}{\gamma-1} k n_\alpha T'_\alpha) &= - \text{div} \left\{ \bar{V}_\alpha (m_\alpha n_\alpha \bar{V}_\alpha \cdot \bar{V}'_\alpha \right. \\
&+ \frac{\gamma}{\gamma-1} k n_\alpha T'_\alpha - \pi'_\alpha) + \bar{V}'_\alpha \left(\frac{1}{2} m_\alpha n_\alpha V_\alpha^2 + \frac{\gamma}{\gamma-1} k n_\alpha T_\alpha - \pi_\alpha \right) + \bar{q}'_\alpha \left. \right\} \\
&+ \frac{e_\alpha}{c} n_\alpha \bar{V}_\alpha \cdot \bar{E} - \bar{V}'_\alpha \cdot \bar{R}'_\alpha - \bar{V}'_\alpha \cdot \bar{R}_\alpha - \bar{V}'_\alpha \cdot \bar{R}'_\alpha + \langle \bar{V}'_\alpha \cdot \bar{R}'_\alpha \rangle + Q'_\alpha \\
&+ \frac{e_\alpha}{c} n_\alpha \left\{ (\bar{V}'_\alpha \cdot \bar{E}) - \langle \bar{V}'_\alpha \cdot \bar{E} \rangle \right\} . \tag{14}
\end{aligned}$$

$$p'_\alpha = k n_\alpha T'_\alpha . \tag{15}$$

These equations must be supplemented by the Maxwell equations for the electromagnetic field:

$$\text{div } \bar{E} = 4\pi c^2 \sigma = 4\pi e c [(n_i - n_e) + (n'_i - n'_e)] , \tag{16}$$

$$\text{div } \bar{B} = 0 , \tag{17}$$

$$\text{curl } \bar{E} = - \frac{\partial \bar{B}}{\partial t} , \tag{18}$$

$$\text{curl } \bar{B} = - \frac{1}{c^2} \frac{\partial \bar{E}}{\partial t} + 4\pi \bar{j}_t \tag{19}$$

where \bar{j}_t denotes the total current and is assumed to be

$$\bar{j}_t = \bar{J} + \bar{j} , \tag{20}$$

$$\text{with } \bar{J} = \frac{e}{c} (n_i \bar{V}_i - n_e \bar{V}_e) , \tag{21}$$

$$\bar{j} = \frac{e}{c} (n'_i \bar{V}'_i - n'_e \bar{V}'_e) . \tag{22}$$

Two equations of charge conservation may be obtained. The equation

$\text{div } \bar{J} = 0$ follows from Eqs. (6). The other one can be obtained from the

Maxwell equations and put in the following form:

$$\frac{c}{e} \frac{\partial}{\partial t} (n_i' - n_e') = \operatorname{div} j \quad . \quad (23)$$

It is clear that Eq. (23) is consistent with Eqs. (12). Eqs. (12) to (19) govern the fluctuations of the flow and the field quantities. For reasons to be given shortly, the equations will be called the "field equations", which are evidently coupled with the "flow equations", Eqs. (6) to (9).

III. REDUCTION OF THE FIELD EQUATIONS

The field equations (12) to (15) will be simplified based on physical reasonings generally adopted for ionized gases, e. g., in Spitzer's treatise (ref. 2). Additional approximations, however, are found necessary in order to reduce these equations to manageable forms

The non-linear inertia terms $\bar{V}_a' \cdot \nabla \bar{V}_a' - \langle \bar{V}_a' \cdot \nabla \bar{V}_a' \rangle$ in Eqs. (13) will be considered first. Assume that the velocity fluctuation V_e' of the electrons is a function of the variable $\{\omega t - k(x)x\}$. (The following argument is likely to be valid under more general conditions, e. g., even when reflected waves are taken into account.) It is seen that the non-linear inertia terms for electrons compared to $\partial \bar{V}_e' / \partial t$ are of the order $V_{eo} \{k + (dk/d \ln x)\} / \omega$, where V_{eo} is a typical electron velocity. When electromagnetic waves propagate into a non-uniform plasma of increasing density in the x direction, $dk/d \ln x$ is a negative quantity. Thus, when consideration is limited to electric fields such that $V_{eo} \ll c$, the speed of light in free space, these inertia terms can be omitted. The terms $\bar{V}_a \cdot \nabla \bar{V}_a' + \bar{V}_a' \cdot \nabla \bar{V}_a$ may be neglected on the same ground, when the macroscopic velocity of the plasma is small compared to c , as in the present work. These terms should be retained for a more detailed consideration of electromagnetic wave propagation in a moving medium.

In addition, the pressure terms $\nabla p_a'$ and the viscous terms $\text{div } \pi_a'$ will also be ignored under the assumption that these effects are relatively small compared to those produced by the electromagnetic field. Thus, the energy equations (14), and the equations of state (15), need not be considered further.

Based on the preceding considerations, the two equations (13) can be considerably simplified. Addition of these two reduced equations yields:

$$\rho \frac{\partial \bar{V}'}{\partial t} = \bar{J} \times \bar{B} + \bar{j} \times \bar{B} - \langle \bar{j} \times \bar{B} \rangle, \quad (24)$$

where $\rho = n_i m_i + n_e m_e$, and $\bar{V}' = (n_i m_i \bar{V}_i' + n_e m_e \bar{V}_e')/\rho$. By subtracting $n_e \partial \bar{V}_e'/\partial t$ from $n_i \partial \bar{V}_i'/\partial t$ and omitting some higher order terms (ref. 2), the following "current" equation is obtained:

$$\frac{m_e c^2}{n_e e^2} \frac{\partial \bar{j}}{\partial t} = \bar{E} + \bar{V} \times \bar{B} - \frac{c}{n_e e} (\bar{J} \times \bar{B}) + \bar{V}' \times \bar{B} - \langle \bar{V}' \times \bar{B} \rangle - \frac{c}{n_e e} (\bar{j} \times \bar{B} - \langle \bar{j} \times \bar{B} \rangle) - \eta \bar{j}, \quad (25)$$

where, in analogous with Spitzer's argument (ref. 2), \bar{R}_e' is assumed to be proportional to \bar{j} , $\bar{V} = (n_i m_i \bar{V}_i + n_e m_e \bar{V}_e)/\rho$, and η is the electrical resistivity of the plasma. A further approximation, as usually adopted when the electromagnetic wave frequency ω is large, is to omit all magnetic terms in Eq. (25). Under this circumstance, it is generally recognized that the perturbed motion of the ions ($\sim \bar{V}'$) is entirely negligible in the presence of a high-frequency electric field. Here stability of the plasma flow is not considered.

The final form of the current equation (25) becomes

$$\frac{m_e c^2}{n_e e^2} \frac{\partial \bar{j}}{\partial t} = \bar{E} - \eta \bar{j}. \quad (26)$$

This equation, together with Eqs. (16) to (23), can be properly called the field equations. The continuity equation and the momentum equation, Eq. (24) for the ions, need not be used, provided the perturbed motion of the ions is disregarded. It should be noted that, when electrostatic waves are considered, the pressure gradient ∇p_e should be restored in Eq. (26).

IV. FLOW EQUATIONS WITHOUT THE EFFECTS OF DIFFUSION AND CHARGE SEPARATION

The flow equations (6), (7), (10, and (11) can be considerably reduced when diffusion and charge separation are disregarded, i. e., $\bar{V}_i = \bar{V}_e = \bar{V}$, and $n_i = n_e = n$. (It is not necessary, at this point, to assume $n_i' = n_e'$.) First, it is noted that by the argument concerning the perturbed motion of protons, the term $\langle V_i'^2 \rangle$ in Eq. (10) and some of the terms containing \bar{V}_i' in Eqs. (7), (10), and (11) can be omitted.

By adding the two momentum equations (7), and omitting terms of the order m_e/m_i , the following equation is obtained:

$$n(m_i \bar{V} \cdot \nabla \bar{V} + m_e \bar{V}_e' \cdot \nabla \bar{V}_e') = - \nabla p + \text{div } \pi + \langle \bar{j} \times \bar{B} \rangle, \quad (27)$$

where $p = p_i + p_e$, and π is the viscous stress tensor of the plasma.

An energy equation for the plasma can be obtained by adding up Eqs. (10) and (11):

$$\text{div} \left\{ \bar{V} \left[\frac{1}{2} n (m_i V^2 + m_e \langle V_e'^2 \rangle) + \frac{\gamma}{\gamma-1} kn (T_i + T_e) - \pi \right] + \bar{q}_e \right\} = \langle \bar{j} \cdot \bar{E} \rangle, \quad (28)$$

where \bar{q}_i is neglected, since it is much smaller than \bar{q}_e (ref. 3). Unless the temperatures of the protons and the electrons are assumed to be equal, an additional energy equation is needed. The energy equation for protons, Eq. (10), with proper simplification can be used for this purpose:

$$\text{div} \left\{ \bar{V} \left(\frac{1}{2} n m_i V^2 + \frac{\gamma}{\gamma-1} n T_i - \pi \right) \right\} = Q_i, \quad (29)$$

where (ref. 1)

$$Q_i = 3n_e k (T_e - T_i) m_e / m_i \tau_e, \quad (30)$$

$$\tau_e = 3m_e^{1/2} (k T_e)^{3/2} / 4(2\pi)^{1/2} e^4 n_e \ln \Lambda, \quad (31)$$

and

$$\Lambda = \left(\frac{T_i T_e}{T_i + T_e} \right)^{3/2} / e^3 n_e^{1/2} \quad . \quad (32)$$

Eq. (28) shows the Ohmic heating of the plasma while, as evident from Eq. (29), the protons are assumed to gain energy only from the electrons by collisions between protons and electrons. An energy equation for the electrons can be obtained by subtracting Eq. (29) from Eq. (28). The resulting equation is:

$$\operatorname{div} \left\{ n \bar{V} \left(\frac{\gamma}{\gamma-1} k T_e + \frac{1}{2} m_e \langle V_e'^2 \rangle \right) + \bar{q}_e \right\} = \langle \bar{j} \cdot \bar{E} \rangle - Q_i , \quad (33)$$

and demonstrates that the Ohmic heating acts directly only on the electrons.

This conclusion is, of course, a direct consequence of the present theory.

However, all these results appear to be physically reasonable approximations.

Hence, when effects of diffusion and charge separation are neglected, the flow equations are the momentum equation, Eq. (27), the energy equations, Eqs. (28) and (29), the equation of state $p = kn(T_e + T_i)$ and the continuity equation $\operatorname{div}(n \bar{V}) = 0$. The terms $\langle V_e'^2 \rangle$ and $\bar{V}_e' \cdot \nabla \bar{V}_e'$ can be expressed in terms of \bar{j} . These flow equations and the field equations given in Section IV form a determinate system of equations. The transport coefficients of viscosity and thermal conductivity for the plasma have been evaluated by Chapman in reference 3, and will not be reproduced here.

V. SOME GENERAL CONSIDERATIONS

A basic property of plasmas is the tendency toward electrical neutrality. Hence, the effects of charge separation can be disregarded inside the plasma except, e.g., when a plasma sheath is considered. However, consideration of charge separation may lead to significant new phenomena in plasma flows, in which strong pressure and temperature gradients exist. This is due to the fact that pressure and temperature gradients tend to promote diffusion and charge separation in the plasma (ref. 4). Such is the case when strong shock waves are present in a plasma. Theoretical analyses (refs. 5, 6) have shown that, when charge separation is considered, for most plasmas the shock structure is no longer of the conventional type with monotonic rise in temperature, pressure, etc. Instead, the flow quantities are found to be oscillatory (in space coordinate) throughout the shock front. This seems to indicate the importance of charge-separation effects in the interaction of electromagnetic waves and shock waves in a plasma.

An analysis of the interaction of electromagnetic waves and shock waves in a non-uniform plasma, including charge-separation effects, is evidently a difficult mathematical problem. It is necessary to resort to the original system of flow equations given in Section II, although some simplifications are possible. Moreover, it may be necessary to consider the induced electrostatic waves (ref. 7) and their coupling with the electromagnetic waves. Further study of this problem is being made.

The present method of analysis has also been applied to the interaction problem for partially-ionized gases. Additional flow equations are obtained

for the neutral particles. The flow equations for the electrons and the ions are modified to allow for the momentum and energy exchanges between the neutral particles and the charged particles. The ionization potential of the neutral particles, which are taken as monatomic, must also be taken into account in the energy equation. Fluctuations of the electron density can also be disregarded in the dynamic equations for partially-ionized gases because, wherever the mean electron density is low, the interaction of the electromagnetic waves and the plasma flow will be very weak. Extension of this theory to include dissociation and other effects can also be made without any essential difficulty, but the problem will become increasingly more complicated.

VI. CONCLUDING REMARKS

A method of analysis and macroscopic equations have been presented for the study of interactions between electromagnetic waves and non-uniform plasma flows. This development is based on hydrodynamic equations given by Shafranov in reference 1. More elaborate systems of such equations have been obtained by Burgers (ref. 8). However, in view of the complexity of the present problem, use of more elaborate but more complicated equations does not appear to be warranted at this time.

The present method of analysis is similar in many aspects to that used for the turbulent shear flows (ref. 9). It has been shown that the physical nature of the interaction problem permits a determinate system of governing equations to be obtained.

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VIII. REFERENCES

1. V. D. Shafranov, "The Structure of Shock Waves in a Plasma," Soviet Physics JETP, Vol. 5, No. 6, p. 1183, December 15, 1957.
2. L. Spitzer, Jr., "Physics of Fully Ionized Gases," Interscience Publishers, p. 20, 1956.
3. S. Chapman, "The Viscosity and Thermal Conductivity of a Completely Ionized Gas," The Astrophysical Journal, Vol. 120, No. 1, p. 151, 1954.
4. S. Chapman and T. G. Cowling, "The Mathematical Theory of Non-Uniform Gases," Cambridge University Press, p. 143, and 255, 1952.
5. O. W. Greenberg and H. K. Sen, "Hydrodynamic Model of Diffusion Effects on Shock Structure in a Plasma," The Physics of Fluids, Vol. 3, No. 3, p. 379, May-June, 1960.
6. O. W. Greenberg and Y. M. Treve, "Shock Wave and Solitary Wave Structure in a Plasma," The Physics of Fluids, Vol. 3, No. 5, p. 769, September-October, 1960.
7. L. Spitzer, Jr., "Physics of Fully-Ionized Gases, Interscience Publishers, p. 58, 1956.
8. J. M. Burgers, "Statistical Plasma Mechanics," Chapter 5 in Plasma Dynamics, edited by F. H. Clauer, Addison-Wesley Co., p. 123-132, 1960.
9. A. A. Townsend, "The Structure of Turbulent Shear Flow," Cambridge University Press, p. 24, 1956.

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SUMMARY

A method of analysis for the interaction between electromagnetic waves and a non-uniform plasma flow is presented. The electromagnetic waves are taken to be in the microwave range. By using a fully-ionized gas as a model, two coupled sets of macroscopic equations are obtained: the flow equations and the field equations. Special consideration is given to the case where diffusion and charge separation can be disregarded. It is shown that both sets of equations can be reduced to useful forms. Physical significance of these equations and extension of this method to include charge separation and partially ionized plasmas are briefly discussed.

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